Meriam 3/1 - 3/5: Kinetics of Particles

Today: sample Problems

- $\vec{F} = m\vec{a}$ vector equation
  valid in all inertial reference frames
  use a free body diagram to solve

- Common Forces:

gravity force $= \dfrac{G m_1 m_2}{r^2}$

$= mg$ on surface of planet

spring force $= -kx$

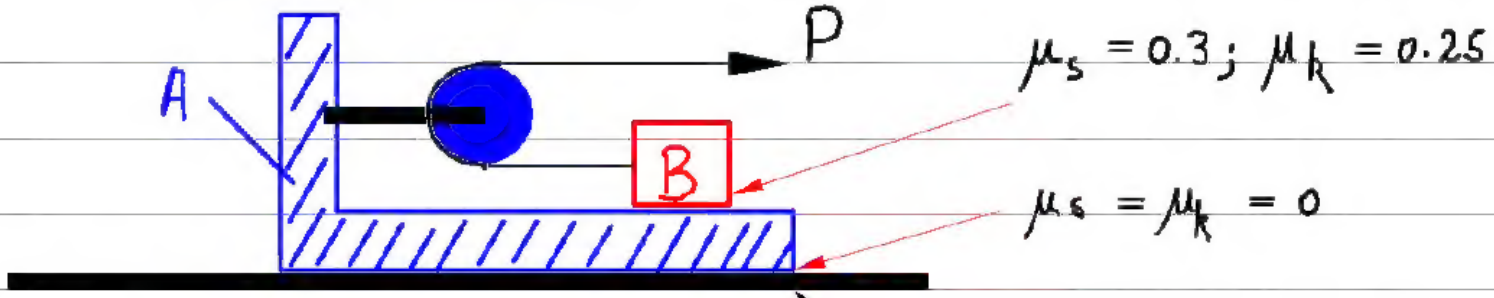
friction force

static: $F \leq \mu_s N$

kinetic: $F = \mu_k N$

E.g.

11(b)



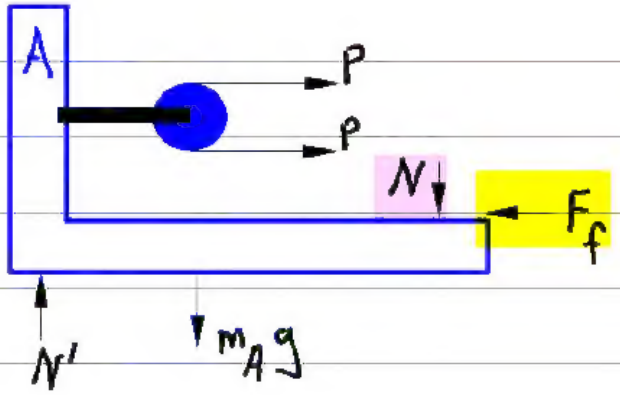
Given: $m_A = 10\ kg$

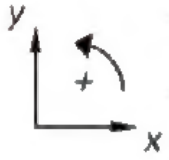
$m_B = 6\ kg$

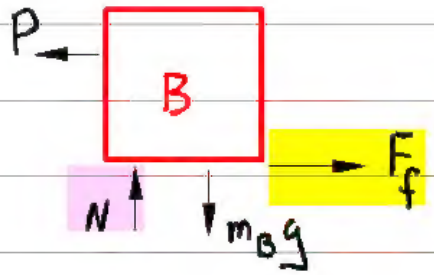
Ground is covered with ice

Find maximum value of P s/t B does not slip on A

Answer: Two free-body diagrams







- Direction of friction opposes slippage
- Highlighted action/reaction forces (Newton's 3rd Law)

(A) $\sum F_x = m_A a_A \rightarrow$ $\boxed{-F_f + 2P = m_A a_A}$

(B) $\sum F_x = m_B a_B \rightarrow$ $\boxed{F_f - P = m_B a_B}$

→ We have: 2 kinetic equations
4 unknowns: a_A, a_B, P, F_f

When B is about to slip on A,

$\boxed{a_A = a_B}$ (kinematic relationship)

$F_f = \mu_s N = \mu_s m_B g = (0.3)(6\,kg)(9.81)$

$\boxed{F_f = 17.7 \text{ Newtons}}$

Solve for the 4 unknowns using the 4 equations:

$P = 12.9$ Newtons

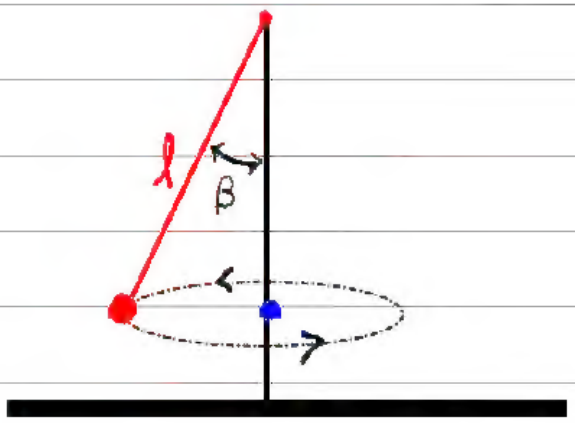
$\vec{a}_A = \vec{a}_B = 0.81\,\hat{i}\ m/s^2$

Friction $F_f = 17.7$ Newtons
(direction in diagram)

Eg. (nt coordinates)

A ball is tied to a flagpole by a red string of length l.

The ball whirls around the flagpole at a constant speed v



Determine angle β and string tension T

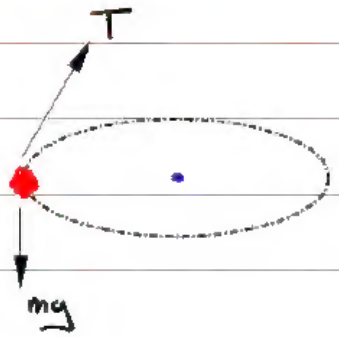
- $l = 1.8$ metens
- $m = 450$ grams
- $V = 4$ m/s

Note:

- Ball is travelling in a circle of radius $r = l \sin\beta$
- Center of circle is indicated by blue dot.
- Use r-θ or n-t coordinates:

$$\vec{v} = 4\,\hat{e}_t \text{ m/s} \quad (\text{or } 4\,\hat{e}_\theta \text{ m/s})$$

Free-body diagram (side view)



$$\Sigma \vec{F} = m\vec{a}$$

1/ Vertical direction: $\Sigma F_{vertical} = m a_{vertical} = 0$

+↑

$$T\cos\beta - mg = 0$$

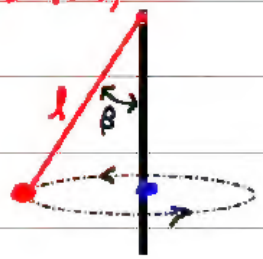
$$\rightarrow \boxed{T\cos\beta = mg} \quad (1)$$

2/ Tangential direction : $\sum F_t = m a_t$

$0 = 0$: no help

3/ Normal direction (towards center • of circular motion)

$$\sum F_n = m a_n$$



$$T \sin\beta = m \frac{v^2}{\rho} \qquad \text{where } \rho = l \sin\beta$$

$$\boxed{T \sin\beta = m \frac{v^2}{l \sin\beta}} \quad (2)$$

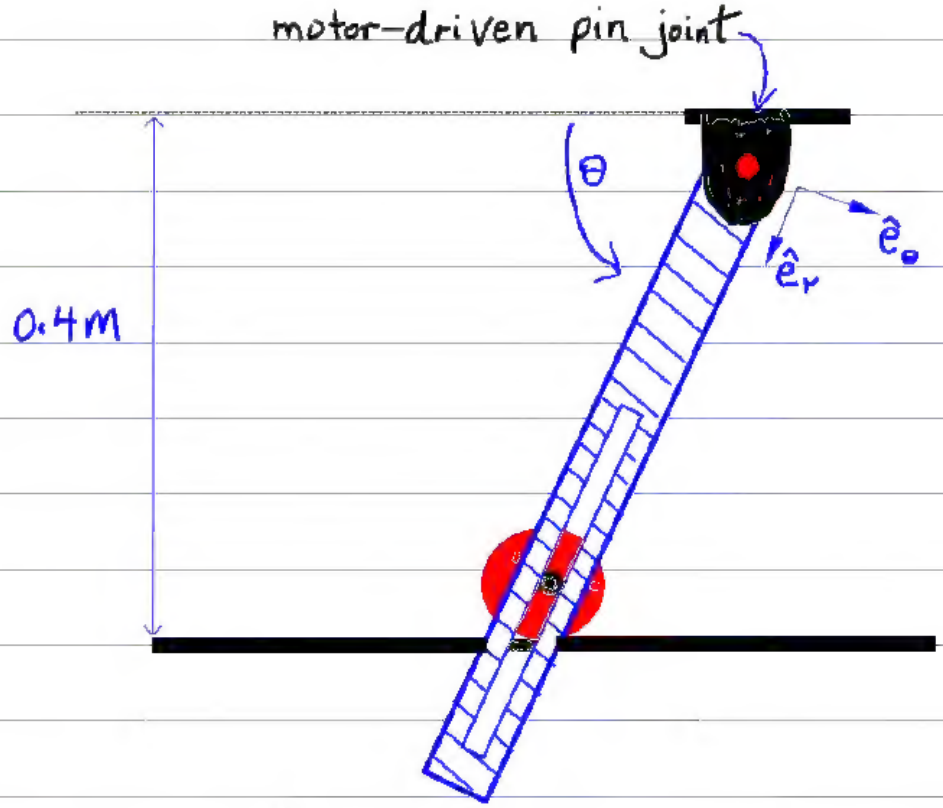
Solve (1) & (2) for β & T

(requires numerical/computer solution)

→ $\beta = 49.9°$

$T = 6.86$ Newtons

Eg.



Given: $\theta = 60^\circ$

$\dot{\theta} = 0.5\ s^{-1}$

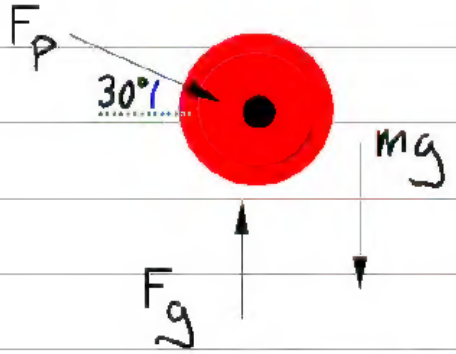
$\ddot{\theta} = 0$

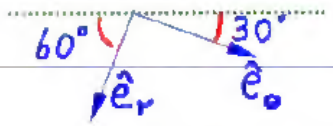
Friction = 0

mass of peg ● = 2 kg

Find force F_p exerted by blue arm on peg ●.

Choose $+\hat{e}_\theta$ direction for peg force F_p





Solution :

We have θ ✓ r ✓

$\dot{\theta}$ ✓ $\dot{r}$ X

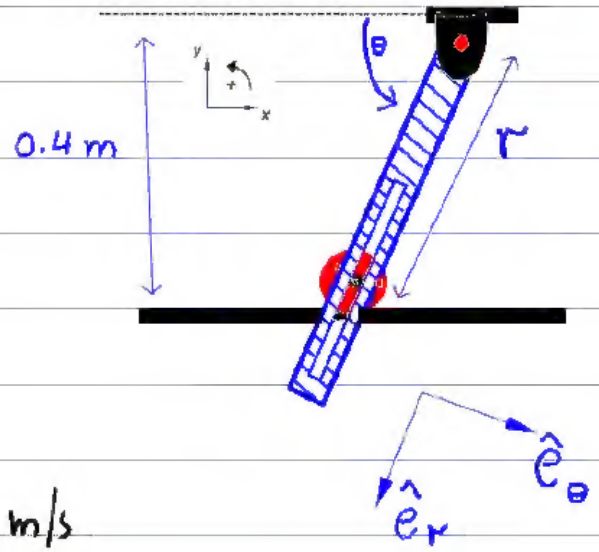
$\ddot{\theta}$ ✓ $\ddot{r}$ X

Get $\dot{r}, \ddot{r} \rightarrow \vec{a} \rightarrow \vec{F} = m\vec{a}$ in r-θ coordinates

$\theta = 60^\circ$ $\qquad r = \frac{0.4}{\sin\theta}$ $\qquad$ 11 (g)

$\dot{\theta} = 0.5\ s^{-1}$

$\ddot{\theta} = 0$



$\frac{d}{dt}\{\sin\theta = 0.4/r\}$

$(\cos\theta)(\dot{\theta}) = -0.4\, r^{-2}\, \dot{r}$ ★

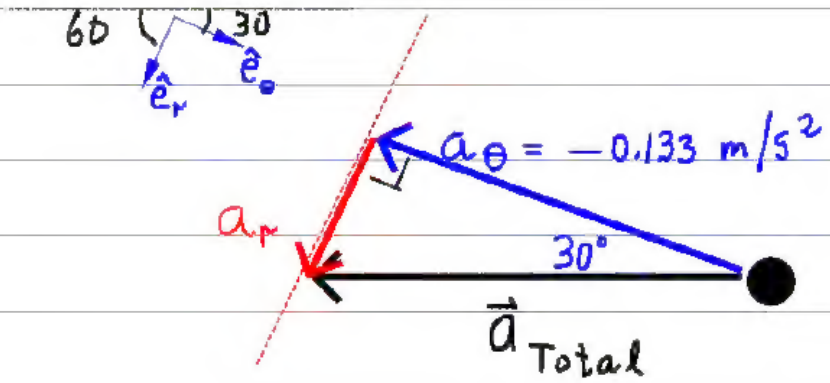
$\rightarrow \dot{r} = \frac{r^2 \dot{\theta} \cos\theta}{-0.4} = -0.133$ m/s

$\rightarrow$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -0.133\ m/s^2$ ($\ddot{\theta}$: 0; $\dot{r}$: −0.133; $\dot{\theta}$: 0.5)

$a_r = \ddot{r} - r\dot{\theta}^2$

We could get $\ddot{r}$ by taking derivative of ★

or Note that total acceleration must be horizontal

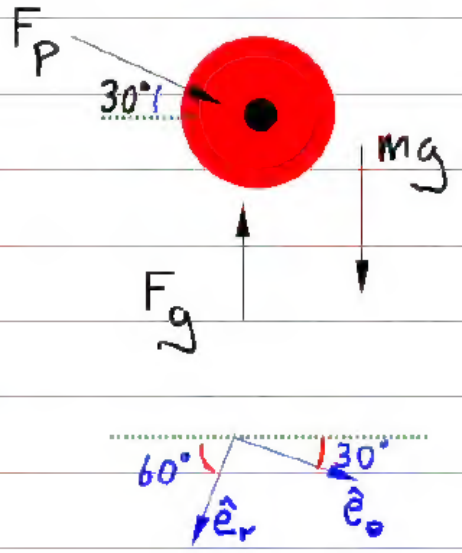


$\rightarrow |a_r| = |a_\theta| \tan 30^\circ = 0.077\ m/s^2$

(Determine +/− signs from diagram)

$$\Sigma F_\theta = m a_\theta$$

$$F_p + mg\cos 60^\circ - F_g \cos 60^\circ = m a_\theta \quad (1)$$



$$\Sigma F_r = m a_r$$

$$-F_g \cos 30 + mg \cos 30^\circ = m a_r \quad (2)$$

Watch +/− signs and angles!

Solve (1) & (2) for

$$F_g = 19.4\,\hat{j} \text{ Newtons}$$

$$F_p = -0.376\,\hat{e}_\theta \text{ Newtons}$$

Summary

- Engineering applications of $\vec{F} = m\vec{a}$
- Applicable in any inertial system: x-y, n-t, r-θ

For next lecture

- Read Meriam 3/6 (work-kinetic energy)